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Semiotic proto- and deutero-addition

1. How are sub-signs added (and subtracted)? According to Berger (1976), semiotic addition is union, e.g.

(2.1) + (2.1) = (2.2)
(2.2) + (2.3) = (2.3), etc.

2. A very interesting suggestion comes from Kaehr (2009), namely rejection, e.g.

(2.1) + (2.2) = (2.3)
(2.1) + (2.3) = (2.1)
(2.2) + (2.3) = (2.1)

However, the problem is that the sum is ambiguous if only one trichtomic value appears in the summands, e.g.

(2.1) + (2.1) = (2.2)? (2.3)?

3. In Toth (2009a), I have already suggested that one could add sub-signs with contextures.

3.1. A first possibility is to build the max both of the trichotomic values and of the contextures, thus max (a.b) and max(i,j), e.g.

$\max((2.1)_1, (2.2)_{1,2}) = (2.2)_2$

3.2. The second possibility is the union building not only from the sub-signs, but also from the contextures, e.g.

$(2.1)_1 + (2.2)_{1,2} = (2.2)_{1,2}$

4. Specifically for semiotic deutero-numbers, we have then for (2.1) and (2.2):

4.1. $(2^1{}_1) + (2^2{}_{1,2}) = (2^2{}_2)$

4.2. $(2^1{}_1) + (2^2{}_{1,2}) = (2^3{}_2)$

5. However, while addition and subtraction of semiotic trito-numbers do not cause any problems (Toth 2009b), the respective operations in the number structure of proto-signs do. Therefore, the question arises how we should best define proto-signs. If we define a proto-sign according to polycontextural theory as a pair of numbers (m:n), where m determines the lenght and n the number of different kenos, then we would get the following semiotisch "proto-matrix":

(2:1) (2:2) (2.2)

(2:2) (2:1) (2:2)

(2:2) (2:2) (2:1)

However, here, most of the non-genuine sub-signs coincide, since from the standpoint of a kenogramm, (2.3) = (1.2) = (0.1), etc. (Kronthaler's "Normal-formoperator).

However, another possibility to write the sub-signs as proto-signs is by interpreting m as semiotic value an n as the occurrence of this semiotic value in a sub-sign. Thus, e.g., in (1.1) 1 is the (triadic) value, and its occurrence is 2, since it is also trochotomic value. But in (1.2), the triadic value 1 occurs only once, and the triadic value 2 occurs only, too, i.e. we get (1:1) (2:1). In this case, the semiotic proto-matrix looks as follows:

(1:2) — (1:1) (2:1) (1:1) (3:1)

(2:1) (1:1) (2:2) — (2:1) (3:1)

(3:1) (1:1) (3:1) (2.:1) (3:2) —

As we see here, besides the genuine sub-signs, all sub-signs are **pairs** of proto-numbers. So, if we want to add (2.1) + (2.2), like above, we get the following strange result:

(2.1) + (2.2) = (2:1) (1:1) + (2.2) — = (2.3) (1.1) (?).

But if we add according to the first maxtrix:

(2:2) + (2:1) = (2:2),

then the sum says not more than it consists again of a single sub-sign with the same triadic value and as trichotomic value max ((2.1), (2.2)), since three different kenos at a length of 2 are not reachable for a "rejective" (*(2.3)). However, this would mean that the successor of (2:2) or (2.1) is identical with the successor of (2:1) or (2.2), namely (2:1) or (2.2), which is apparently nonsense.

Thus, let us attempt at adding proto-signs starting with the notation of successor.

(1:2) —	(1:1) (2:1)	(1:1) (3:1)
(2:1) (1:1)	(2:2) —	(2:1) (3:1)
(3:1) (1:1)	(3:1) (2.:1)	(3:2) —

In a "natural" way, a proto-sign (m:n) has basically two successors: 1) (m+1):n, and 2) m: (n+1). Now we see from the matrix

(1.2) — → (1:1) (2:1)

(1.2) — → (2:1) (1:1)

(1:2) — → (2:2)

"Geomerically" these are thus the three possible successors of (1:2) —. That means, that the successor structures are:

(a.b) — → (a:a) (b:a)

(a:b) — → (b:a) (a:a)

(a:b) — → (b:b),

or generally with a successor operator S(a) = (a+1):

$(a.(a+1)) \longrightarrow (a{:}a)\ ((a+1){:}a)$

$(a{:}(a+1)) \longrightarrow ((a+1){:}a)\ (a{:}a)$

$(a{:}(a+1)) \longrightarrow ((a+1){:}(a+1))$.

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